Math 121   **Test #2**   **Spring 2007**

1. (26 pts) For the sample

$$32 \quad 41 \quad 36 \quad 32 \quad 37 \quad 32 \quad 35 \quad 38 \quad 25 \quad 42 \quad 35$$

   do the following

   (a) (6 pts) Find the mean.

   (b) (6 pts) Find the standard deviation.

   (c) (6 pts) Find the 5-number summary of the data.

   (d) (8 pts) Draw a modified boxplot of the data.

2. (6 pts) Using the sample in problem 1, find the 85th percentile. You may use the formula $r = 1 + \frac{p}{100}(n-1)$.

3. (9 pts) Find the following areas under the standard normal curve.

   (a) The area to the left of -1.65.

   (b) The area between -1.65 and 1.85.

   (c) The area to the right of 1.85.

4. (4 pts) Find the 85th percentile of the standard normal distribution.

5. (12 pts) Let the distribution of the random variable X be $N(30, 5)$. Find the following.

   (a) The likelihood that $X < 25$.

   (b) The likelihood that $22 < X < 32$.

   (c) The likelihood that $X > 44$.

   (d) The 45th percentile of X.

6. (9 pts) Suppose that waiting times, in minutes, at a toll booth are uniform on the interval from 0 to 6. Then the average of two randomly selected waiting times has the following distribution.
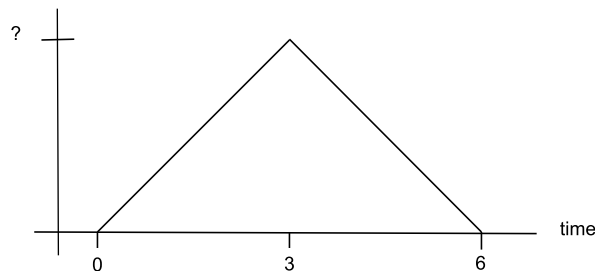
(a) (3 pts) What is the height of the graph at the peak?

(b) (6 pts) What is the probability that a randomly selected average is less than 1 minute?

7. (10 pts) Let the random variable X represent the time required to perform a specified task. Consider the following two hypotheses whose drawings are shown below:

H_0: X is uniform between 0 and 5.

H_1: X is uniform between 2 and 7.

A choice between the two hypotheses will be made based on a single value of X. The decision rule is to reject H_0 if the observed value of X is at least 4.

(a) (5 pts) What is the value of α?

(b) (5 pts) What is the value of β?
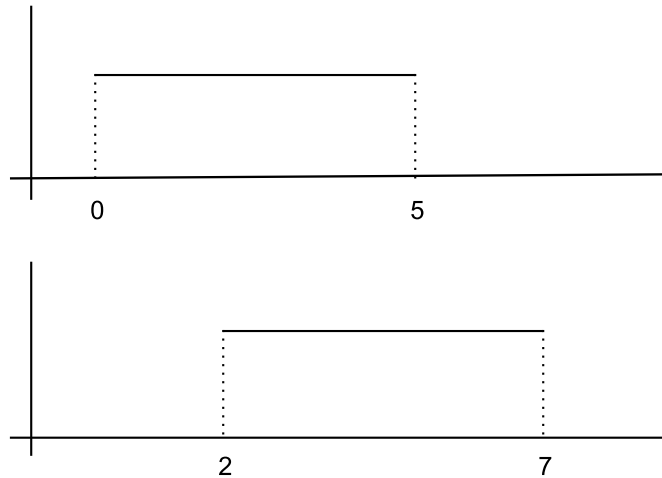
8. (12 pts) Suppose that in a population of commuters, 1/3 of them commute to work by public transportation and 2/3 of them commute to work by private vehicle. Using a sample size of $n = 2$, let $\hat{p}$ be the sample proportion of workers who commute by public transportation.

(a) (3 pts) What are the possible values of $\hat{p}$.

(b) (6 pts) Find the sampling distribution of $\hat{p}$. That is, find the likelihood of each value of $\hat{p}$. You may want to draw a tree diagram to help you.

(c) (3 pts) What are the mean and standard deviation of the distribution that you found in part (a)?

9. (12 pts) Suppose that a random variable X represents waiting times that have a uniform distribution $U(0,5)$. Then it turns out that $\mu = 2.5$ and $\sigma = 1.443$. Now let the random variable $\overline{X}$ represent the average of 100 randomly selected values of X.

   (a) (4 pts) Describe precisely the sampling distribution of $\overline{X}$.

   (b) (8 pts) Find the probability that a randomly selected sample of 100 values of X will have an average that is between 2.25 and 2.75.